Growing Patterns and Sequences

Reporting Category	Patterns, Functions, and Algebra
Topic	Identifying and extending arithmetic and geometric sequences
Primary SOL	6.17 The student will identify and extend geometric and arithmetic sequences.

Materials

- Toothpicks
- Toothpick Patterns recording sheet (attached)
- Inch tiles
- Inch Tile Patterns recording sheet (attached)
- Comparing Arithmetic Sequences to Geometric Sequences handout (attached)
- Calculators

Vocabulary

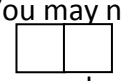
square, rectangle, pattern (earlier grades)

exponent (6.5)

growing pattern, arithmetic sequence, geometric sequence (6.17)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)
Note: This lesson will take more than one day to complete. You may want to spend a day on each type of sequence and a third day comparing the two types of sequences.

Part A: Arithmetic Sequence

1. Introduce the lesson by explaining that students will be using tools to create sequences of squares and analyze the growing patterns that emerge. Ask students to predict what a growing pattern is and what might be happening in a growing pattern. Have them share their ideas.
2. Give each student a pile of toothpicks, and ask each student to create a square, using the fewest number of toothpicks possible. Ask students how many toothpicks they used. (4)
3. Distribute copies of the Toothpick Patterns recording sheet. Explain that the table will be used to record data as students create additional squares. (Make sure students understand how to read an input/output table.) Display a large-scale version of the table, and fill in the data for one square.
4. Ask each student to make a rectangle consisting of two connected squares, using the fewest number of toothpicks possible. You may need to model this step to ensure that all students create the same type of figure.  Ask students what they notice. Have them count and record in their tables the number of toothpicks (7). Record 7 toothpicks in the display table.

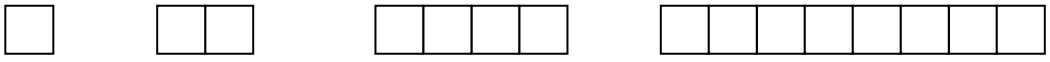
5. Ask each student to build onto their figure to create three squares.  Again, ask them what they notice. Have them count and record in their tables the number of toothpicks (10). Record 10 toothpicks in the display table.
6. Ask students to continue building squares onto their figures, creating each square with the fewest number of toothpicks possible. Have them record their data in their tables as they work. Tell them to look for a pattern to answer the questions on the recording sheet.
7. After students have completed their tables, have the class complete the display table and then discuss the growing pattern. Have students describe the pattern and how it changes from term to term. Your completed table should look like this:

Number of Squares	Number of Toothpicks
1	4
2	7
3	10
4	13
5	16
6	19
7	22

8. Ask students to discuss how they could determine the number of toothpicks needed to make 12 squares. Ask how they could determine the number needed to make 15 squares. Allow them time to explore these questions, either individually or with partners. Have them justify how they know they are correct.
9. Display the sequence of numbers 6, 9, 12, 15, 18, …, and have students think about what the next three terms in the sequence are. Allow students time to work, and then have them share their responses, including their strategies. Have partners compare this sequence to the sequence created using the toothpicks. Have them share their responses.
10. Display the sequence of numbers 5, 7, 9, 11, 13, …, and have students extend the sequence and then discuss how they were able to determine the next terms in the sequence. Have partners compare this sequence to the two previous sequences.
11. Ask students what they notice about how these patterns grow. Be sure they see that in these sequences, **the same amount is added to each term to obtain the next term in the sequence.** Tell students that a number pattern that grows in such a way is called an **arithmetic sequence.** Write "Arithmetic Sequence" above the display table containing the toothpick pattern data, and have students write "Arithmetic Sequence" above their own tables.

Part B: Geometric Sequence

1. Distribute copies of the Inch Tile Patterns recording sheet. Explain that the table will again be used to record data as students create a new growing pattern, using the tiles. They will again be making rectangles out of squares, but this time they will record the number of tiles used rather than the number of toothpicks. Display a large-scale version of the table, and fill in the data for the first figure.
2. Model how to create the first four rectangles in this pattern, using inch tiles. (You may want to create larger models of inch tiles to use for demonstration.)



Ask students to count and record in their tables the number of tiles in each of these figures. Record 1, 2, 4, and 8 tiles in the display table.

3. Have students compare this pattern to the pattern created with the toothpicks, asking whether this pattern is growing in the same way as the toothpick pattern.
4. Give each student a pile of inch tiles, and have each student use the tiles to construct the next figure in the pattern. (The next figure should consist of 16 tiles.) Have students share their the number of tiles they used and fill in their tables. Ask them to explain how they know the next figure consists of 16 tiles.
5. Ask students to continue building onto their figures, following the pattern that has been established. Have them record their data in their tables as they work. Tell them to think about the pattern to answer the questions on the recording sheet.
6. After students have completed their tables, have the class complete the display table and then discuss the growing pattern. Have students describe the pattern and how it changes from term to term. Your completed table should look like this:

Figure	Number of Inch Tiles
1	1
2	2
3	4
4	8
5	16
6	32
7	64

7. Display the sequence of numbers 1, 5, 25, 125, 625, …, and have students think about what the next three terms in the sequence are. Allow time for students to work, and then have them share their responses, including their strategies. Have partners compare this sequence to the sequence created using the tiles. Have them share their responses.
8. Display the sequence of numbers 1, 3, 9, 27, 81, …, and have students extend the sequence and then discuss how they were able to determine the next terms in the sequence. Have partners compare this sequence to the two previous sequences.
9. Ask students what they notice about how these patterns grow. Be sure they see that in these sequences, **each term is multiplied by the same amount to obtain the next term in the sequence.** Tell students that a number pattern that grows in such a way is called a **geometric sequence.** Write "Geometric Sequence" above the display table containing the inch tile pattern data, and have students write "Geometric Sequence" above their own tables.

Part C: Comparing Arithmetic Sequences to Geometric Sequences

1. Display these arithmetic sequences and geometric sequences.

Arithmetic Sequence	Geometric Sequence
6, 9, 12, 15, 18 …	1, 5, 25, 125, 625 …
5, 7, 9, 11, 13 …	1, 3, 9, 27, 81 …

2. Show students how to take the data from the tables and write the sequences in a list. Have students look at the two types of sequences and discuss with partners how they are the same and how they are different.
3. Distribute calculators and copies of the Comparing Arithmetic Sequences to Geometric Sequences handout. Use this opportunity to review using an input/output table. Have students work with partners to complete the handout. Allow students access to toothpicks, inch tiles, or other manipulatives they may use to help them extend the patterns on the handouts.
4. After students have had time to complete the handouts, hold a class discussion to compare arithmetic sequences to geometric sequences. Record student ideas. Students should understand how multiplying by 5, for example, is like adding 5 and that raising a number to a power is like multiplying.

Assessment

- **Questions**
  - What is the difference between the toothpick sequence (arithmetic sequence) and the inch tile sequence (geometric sequence)?
- **Journal/Writing Prompts**
  - Choose one of the patterns explored during this lesson. Describe the pattern, and explain your strategy for extending the pattern.
  - Describe an example of a real-world arithmetic sequence.
  - Describe an example of a real-world geometric sequence.
- **Other**
  - Use the Toothpick Patterns recording sheet as an assessment.
  - Use the Inch Tile Patterns recording sheet as an assessment.

Extensions and Connections (for all students)

- In Part A of the lesson, ask students to describe a rule for predicting the number of toothpicks needed to make *n* squares. There are several approaches in generalizing the problem. Here is one example of students' thinking. Some students may have already noticed that for each new figure, three toothpicks are added to the number of toothpicks in the preceding figure. The figure number (*n*) multiplied by 3 plus 1 equals the number of toothpicks (*t*) needed to construct a figure with *n* squares. If students are ready, you may want to introduce how to translate the verbal rule into a mathematical expression (i.e., $t = 3n + 1$).
- In Part B of the lesson, ask students to describe a rule for predicting the number of inch tiles (*I*) in figure *n*. Again there are several approaches in generalizing the problem. Here is one example of students' thinking. Some students may have already noticed that for each

new figure, the number of squares in the preceding figure is doubled. The previous figure's number should be used to determine the number of times to multiply 2 (i.e., $I = 2^{n-1}$).
- Have students make their own patterns for partners to solve.

Strategies for Differentiation
- In Part I, provide a larger manipulative, such as, popsicle sticks, base-10 sticks, straws, or pretzel rods, to students instead of toothpicks.
- Allow students to use calculators to help them determine the differences between terms in the sequences.
- Allow students access to tools, such as toothpicks or inch tiles, throughout the entire lesson.

Toothpick Patterns

Name _____ **Date** _____

Number of Squares	Number of Toothpicks
1	**4**
2	
3	
4	
5	
6	
7	
8	
9	
10	

Describe the pattern, using words.

Describe how you find the next term in the pattern.

Inch Tile Patterns

Name _____ **Date** _____

Figure	Number of Inch Tiles
1	**1**
2	
3	
4	
5	
6	
7	
8	
9	
10	

Describe the pattern, using words.

Describe how you find the next term in the pattern.

Comparing Arithmetic Sequences to Geometric Sequences

Name _____ **Date** _____

Arithmetic Sequences

Rule: 5*n*	
Input *(n)*	**Output**
1	5
2	10
3	15
4	
5	
6	
7	

Rule: 4*n*	
Input *(n)*	**Output**
1	4
2	8
3	12
4	
5	
6	
7	

Geometric Sequences

Rule: 4^n	
Input *(n)*	**Output**
1	4
2	16
3	64
4	
5	
6	
7	

Rule: 5^n	
Input *(n)*	**Output**
1	5
2	25
3	125
4	
5	
6	
7	

What is the same about the arithmetic sequences and the geometric sequences?

What is different about the arithmetic sequences and the geometric sequences?